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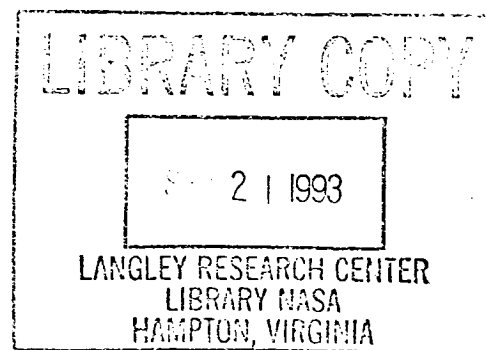
# Kinematics and Constraints Associated with Swashplate Blade Pitch Control

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March 1993



National Aeronautics and  
Space Administration



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## CONTENTS

	Page
NOMENCLATURE.....	iv
SUMMARY .....	1
1.0 INTRODUCTION.....	2
2.0 TECHNICAL.....	2
2.1 Rotating System Fourier Coefficients.....	2
2.2 Nonrotating System Fourier Coefficients.....	3
2.3 Individual Actuator Stroke .....	5
2.4 Blade Pitch Expressed as a Function of Blade Azimuth and Actuator Stroke .....	5
2.4.1 Canonical Swashplate Plane Equation .....	6
2.4.2 Vertical Position of a General Field Point.....	8
2.4.2.1 Vertical position of a general field point expressed in the nonrotating system.....	9
2.4.2.2 Vertical position of a general field point expressed in the rotating system.....	10
2.4.2.3 Blade pitch.....	16
2.5 Rate Control.....	18
2.5.1 Rates for the Form $y = A \sin \omega t + B \cos \omega t$ .....	18
2.5.2 Rates for the Form $y = y_0 + C \sin (\omega t + \gamma)$ .....	19
2.6 Power.....	19
2.6.1 Nonrotating Subsystems.....	21
2.6.2 Rotating Subsystems.....	22
3.0 RESULTS .....	24
4.0 REFERENCES.....	24
APPENDIX A Rule for the Linear Combination of Monofrequency Sinusoidal Terms.....	25

## NOMENCLATURE

$A$	general coefficient for the $\sin(\omega t)$ term
$A_{COL}$	least upper bound (l.u.b) for the Nonrotating System Collective Fourier coefficient magnitude constraint function
$A_{LAT}$	least upper bound (l.u.b.) for the Nonrotating System Lateral Cyclic Fourier coefficient magnitude constraint function
$A_{LONG}$	least upper bound (l.u.b.) for the Nonrotating System Longitudinal Cyclic Fourier coefficient magnitude constraint function
$A_{N-1}, A_N, A_{N+1}$	least upper bounds (l.u.b.s) for the Rotating System $N-1$ , $N$ , and $N+1$ per rev Fourier coefficient magnitude constraint functions, respectively, defined in section 2.3
$A_R$	general least upper bound (l.u.b) for a rate control magnitude constraint function
$A_1, A_2, A_3$	least upper bounds (l.u.b.s) for the first, second, and third actuator vertical excursions, respectively, defined in section 2.3
$A_1, A_2, A_3$	coefficients appearing in equation (14) as defined in equations (15) of section 2.4.2
$A_0, A_1, A_2, A_3, A_4$	general coefficients of the first product term appearing in equation (53) of section 2.6.2
$B$	general coefficient for the $\cos(\omega t)$ term
$B_1, B_2, B_3$	coefficients appearing in equation (14) as defined in equations (15) of section 2.4.2
$B_1, B_2, B_3, B_4$	general coefficients of the second product term appearing in equation (53) of section 2.6.2
$B_1, B_2, B_3, B_4, B_5$	general coefficients appearing in equation (48) of section 2.6.1
$C$	general coefficient for the $\sin(\omega t + \gamma)$ term
$C_x, C_y, C_z$	cofactors of the $(x - x_1)$ , $(y - y_1)$ , and $(z - z_1)$ elements in the $\mathcal{D}$ matrix, respectively
$C_0, C_1, C_2$	general coefficient appearing in equation (49) of section 2.6.1
$C_1, C_2, C_3$	amplitudes of the first, second, and third actuator vertical excursions, respectively, defined in section 2.3
$C_1, C_2, C_3$	coefficients appearing in equation (14) as defined in equations (15) of section 2.4.2
$C_1, C_2, \dots, C_{14}$	general coefficients appearing in equation (55) of section 2.6.2
$\mathcal{D}$	matrix associated with the swashplate plane equation defined by equation (2) of section 2.4.1
$D$	determinant of the $\mathcal{R}$ matrix defined by equation (6) in section 2.4.1
$D$	the constant of proportionality for the kinetic energy $T$ appearing in equation (43) in section 2.6
$D_x, D_y, D_z$	constants appearing in the Canonical form of the swashplate plane equation [eq. (5) of sec. 2.4.1] which equal $D/C_x$ , $D/C_y$ , and $D/C_z$ , respectively where $D = \text{Det}(\mathcal{R})$

$D_0, D_1, D_2$	general coefficients appearing in equation (50) of section 2.6.1
$D_0, D_1, D_2, \dots, D_{12}$	general coefficients appearing in equations (58) and (59) of section 2.6.2
$E$	total mechanical energy defined by equation (38) in section 2.6
$E_1, E_2, E_3$	amplitudes of the first, second, and third actuator vertical excursions, respectively, appearing in equation (17) of section 2.4.2.1
$F$	general force unit
$F$	constant appearing in equation (42) of section 2.6 which equals one-half of the effective spring constant of the mechanical system
$F_s$	force acting on the mechanical system due to the effective spring constant appearing in equation (39) of section 2.6
$F_1, F_2, \dots, F_L$	general coefficients of the $\sin(\theta + \phi_{s_k})$ terms appearing in equations (A-1) and (A-10) of appendix A
$G$	constant appearing in equation (42) of section 2.6 which is the reaction to the spring constant at $z_0$
$g.l.b.$	greatest lower bound
$G_1, G_2, \dots, G_m$	general coefficients of the $\cos(\theta + \phi_{c_k})$ terms appearing in equation (A-10) of appendix A
$H$	constant appearing in equation (42) of section 2.6 which represents a reference energy equal to $K(z_0 z_{00} - \frac{1}{2} z_{00}^2)$
$H_c$	constant defined by equation (A-4) or (A-13) of appendix A which equals the sum of the phase angle cosine terms
$HHC$	higher harmonic control
$H_s$	constant defined by equation (A-4) or (A-13) of appendix A which equals the sum of the phase angle sine terms
$i$	blade number; $i = 1, 2, \dots, N$
$j$	index number for the $D_0, D_1, D_2, \dots, D_{12}$ general amplitudes appearing in equations (58) and (59) of section 2.6.2
$K$	effective spring constant of the mechanical system with units of $F/l = m/t^2$ appearing in equation (39) of section 2.6
$k$	actuator number; $k = 1, 2, 3$
$K_E$	constant of proportionality that relates the mechanical power ( $P_M$ ) to the time rate-of-change of the total mechanical energy ( $E$ ) appearing in equation (37) of section 2.6
$K_p$	constant of proportionality that relates the hydraulic power ( $P_H$ ) to the mechanical power ( $P_M$ ) appearing in equation (36) of section 2.6
$l$	general length unit
$l$	pitch link connecting arm length appearing in equation (29) of section 2.4.2.3
$l.u.b.$	least upper bound
$m$	number of oscillations per revolution in section 2.2
$m$	general mass unit
$N$	number of blades
$O$	origin of the $X - Y - Z$ coordinate system defined in section 2.3 (see fig. 1)

$P$	general point on the swashplate plane (see fig. 1)
$P_1, P_2, P_3$	points of intersection of the swashplate plane with the vertical axis of the first, second, or third actuator, respectively (see fig. 1)
$P_H$	hydraulic power requirement defined by equation (36) of section 2.6
$P_M$	generated mechanical power defined by equation (37) of section 2.6
$\mathcal{R}$	matrix associated with the canonical swashplate plane equation [eq. (5) of sec. 2.4.1]
$R$	radius vector from the origin (O) to a general point (P) on the swashplate plane (see fig. 1)
$r$	radius magnitude of the right circular cylinder described by the rotating pitch links (see fig. 2)
$sc(\cdot)$	composite trigonometric function which denotes either $\sin(\cdot)$ or $\cos(\cdot)$
$T$	kinetic energy of the mechanical system appearing in equation (38) of section 2.6
$t$	general time unit
$t$	time
$U$	control 6-vector $[E_1 \ E_2 \ E_3 \ \phi_1 \ \phi_2 \ \phi_3]^T$ defined by equation (18) of section 2.4.2.1
$V$	potential energy of the mechanical system appearing in equation (38) of section 2.6
$V_1, V_2$	potential energy of the mechanical system at states 1 or 2, respectively appearing in equation (39) of section 2.6
$w$	general sum of monofrequency sinusoidal terms defined by equation (A-1) or (A-11) of appendix A
$X$	first coordinate axis of the $X - Y - Z$ coordinate system defined in section 2.3 (see fig. 1)
$x, x_1, x_2, x_3$	component of point $P, P_1, P_2$ , or $P_3$ along the $X$ -axis (see fig. 1)
$Y$	second coordinate axis of the $X - Y - Z$ coordinate system defined in section 2.3 (see fig. 1)
$y$	general sum of monofrequency sinusoidal terms of defined by equation (30) of section 2.5.1 and by the equation (33) of section 2.5.2
$y, y_1, y_2, y_3$	component of point $P, P_1, P_2$ , or $P_3$ along the $Y$ -axis (see fig. 1)
$y_0$	average value of the sinusoidal $y$ defined in section 2.5
$Z$	third coordinate axis of the $X - Y - Z$ coordinate system defined in section 2.3 (see fig. 1)
$z, z_1, z_2, z_3$	component of point $P, P_1, P_2$ , or $P_3$ along the $Z$ -axis (see fig. 1)
$z_0$	reference actuator length defined in sections 2.3 and 2.6.1
$z_0$	vertical displacement ( $z$ ) at which the effective spring force is zero in equation (4) of section 2.6
$z_{00}$	reference value of $z$ which corresponds to the reference value of $\theta, \theta_0$ in equation (29) of section 2.4.2.3
$z_{00}$	reference value of $z$ at which $V_1$ is defined in equation (39) of section 2.6
$z_{01}, z_{02}, z_{03}$	average value of the sinusoidal $z_1, z_2$ , or $z_3$ , respectively defined in sections 2.3 and 2.6.1



$\alpha$	general angular argument
$\beta$	general phase angle
$\beta$	phase angle equivalent of the $H_s$ and $H_c$ coefficients appearing in equation (A-5) or (A-12) of appendix A
$\gamma$	general phase angle
$\gamma$	angular argument equal to either $(N - 1)\Omega t$ , $N\Omega t$ , or $(N + 1)\Omega t$ in section 2.1
$\gamma$	phase angle equivalent of the $A$ and $B$ coefficients appearing in equation (30) of section 2.5.1
$\zeta_1, \zeta_2$	phase angles appearing in equation (49) of section 2.6.1
$\theta$	general Fourier coefficients
$\theta$	blade pitch angle defined in section 2.4.2.3 (see fig. 3)
$\theta$	general angular argument appearing in equations (A-1) and (A-10) of appendix A
$\theta_c$	general Fourier coefficients for cosine terms in section 2.1
$\theta_{COL}$	general Fourier coefficients for the collective terms in section 2.2
$\theta_{LAT}$	general Fourier coefficients for the lateral cyclic terms in section 2.2
$\theta_{LONG}$	general Fourier coefficients for the longitudinal cyclic terms in section 2.2
$\theta_{N-1}, \theta_N, \theta_{N+1}$	general Fourier coefficients for the $N-1$ , $N$ , $N+1$ harmonic terms in section 2.1
$\theta_s$	general Fourier coefficients for sine terms in section 2.1
$\theta_0$	reference value of the blade pitch angle defined in section 2.4.2.3 (see fig. 3)
$\xi$	dummy integration variable corresponding to $z$ in equation (40) of section 2.6
$\phi_{c1}, \phi_{c2}, \phi_{cM}$	phase angles of the cosine terms of in equation (A-10) of appendix A
$\phi_s$	phase angle used to define the relation between the rotating system and the nonrotating system in equation (19) of section 2.4.2.2
$\phi_{s1}, \phi_{s2}, \phi_{sL}$	phase angles of the sine terms in equation (A-10) of appendix A
$\phi_1, \phi_2, \phi_3$	phase angle of the sinusoidal motion of the first, second, and third actuators, respectively in equations (16) and (20) of sections 2.4.2.1 and 2.4.2.2, respectively
$\psi$	phase angle equivalent to the $\theta_s$ and $\theta_c$ coefficients appearing in the equation defining $y$ in section 2.1
$\psi_1, \psi_2$	phase angles appearing in equation (50) of section 2.6.1
$\omega$	general frequency coefficient of $t$ in sinusoidal terms
$\Omega$	rotor rotation rate
<b>Subscripts</b>	
$c$	cosine terms
$COL$	collective terms in section 2.2
$E$	energy in section 2.6
$H$	hydraulic processes in section 2.6
$i$	blade number; $i = 1, 2, \dots, N$
$j$	index for the $D_j$ coefficients appearing in equations (58) and (59) of section 2.6.2
$k$	actuator number; $k = 1, 2, 3$

$L$	number of monofrequency sine terms in equation (A-10) of appendix A
$LAT$	lateral cyclic terms in section 2.2
$LONG$	longitudinal cyclic terms in section 2.2
$M$	mechanical power and mechanical processes in section 2.6
$M$	number of monofrequency cosine terms in equation (A-10) of appendix A
$MAX$	least upper bound (l.u.b.) for a constraint function
$N$	number of blades
$P$	power in section 2.6
$s$	sine terms
$s$	rotating system, pitch links, blades, etc., in section 2.4.2.2
$x$	( $x-x_1$ ) element in matrix $\mathcal{D}$ in section 2.4.1
$y$	( $y-y_1$ ) element in matrix $\mathcal{D}$ in section 2.4.1
$z$	( $z-z_1$ ) element in matrix $\mathcal{D}$ in section 2.4.1
0	reference actuator length
0	vertical displacement of which the effective spring force is zero in section 2.6
00	reference value of $z$ corresponding to the reference value of $\theta$ , $\theta_0$
0, 1, 2	index values for the $C_0$ , $C_1$ , and $C_2$ coefficients appearing in equation (49) of section 2.6.1, and for the $D_0$ , $D_1$ , and $D_2$ coefficients appearing in equation (50) of section 2.6.1
0, 1, 2, $\dots$ , 12	values for the $j$ index of the $D_j$ coefficients appearing in equations (58) and (59) of section 2.6.2
1, 2	states at which the potential energy of the mechanical system is defined
1, 2	index values for the $\zeta_1$ and $\zeta_2$ phase angles appearing in equation (49) of section 2.6.1, and for the $\Psi_1$ and $\Psi_2$ phase angles appearing in equation (50) of section 2.6.1
1, 2, 3	index values for the actuators
1, 2, 3, 4, 5	index values for the $B_1$ , $B_2$ , $B_3$ , $B_4$ , and $B_5$ coefficients appearing in equation (48) of section 2.6.1
1, 2, 3, $\dots$ , 14	index values for the $C_1$ , $C_2$ , $C_3$ , $\dots$ , $C_{14}$ coefficients appearing in equation (55) of section 2.6.2

## SUMMARY

An important class of techniques to reduce helicopter vibration is based on using a Higher Harmonic controller to optimally define the Higher Harmonic blade pitch. These techniques typically require solution of a general optimization problem requiring the determination of a control vector which minimizes a performance index where functions of the control vector are subject to inequality constraints. Six possible constraint functions associated with swashplate blade pitch control were identified and defined. These functions constrain: 1) blade pitch Fourier Coefficients expressed in the Rotating System, 2) blade pitch Fourier Coefficients expressed in the Nonrotating System, 3) stroke of the individual actuators expressed in the Nonrotating System, 4) blade pitch expressed as a function of blade azimuth and actuator stroke, 5) time rate-of-change of the aforementioned parameters, and 6) required actuator power. The aforementioned constraints and the associated kinematics of swashplate blade pitch control by means of the strokes of the individual actuators are documented herein.

## 1.0 INTRODUCTION

An important class of techniques to reduce helicopter vibration is based on using a Higher Harmonic controller to optimally define the Higher Harmonic blade pitch. These techniques typically require solution of a general optimization problem requiring the determination of a control vector which minimizes a performance index where functions of the control vector are subject to inequality constraints (see sec. 2.3.1 of ref. 1). When solving these problems, it is extremely important to remember that the "truthfulness" or "correctness" of the solution depends not only on the validity of the mathematical methods employed to obtain the solution, but also on the "truthfulness" or "correctness" of the problem definition. In deductive mathematics, as in the case classical Aristotelian deductive logic, it is necessary that BOTH the syllogistic deductive reasoning be VALID and the assumed propositions be TRUE in order to assert that the conclusion is TRUE.

The "truthfulness of the propositions" is often overlooked or ignored when attempting to solve difficult mathematical optimization problems with sometimes unfortunate results. Obviously, approximations to "truthful" propositions can be required in order to reduce the problem to a tractable form, but oftentimes these approximations are made because of misunderstandings of what the problem really is (i.e., the real definition of the performance index and constraints). Concern about the correctness of the constraint formulation for the helicopter vibration minimization problem prompted the analysis documented herein.

## 2.0 TECHNICAL

Six possible constraint functions associated with swashplate blade pitch control were identified and defined. These functions constrain: 1) blade pitch Fourier Coefficients expressed in the Rotating System, 2) blade pitch Fourier Coefficients expressed in the Nonrotating System, 3) stroke of the individual actuators expressed in the Nonrotating System, 4) blade pitch expressed as a function of blade azimuth and actuator stroke, 5) time rate-of-change of the aforementioned parameters, and 6) required actuator power. The aforementioned constraints and the kinematics of swashplate blade pitch control referenced to the individual strokes of the actuators in the nonrotating system are described in the following subsections.

### 2.1 Rotating System Fourier Coefficients

If given the Fourier Coefficients for an entity expressed in the Rotating System, say  $\theta_{(N-1)c}$ ,  $\theta_{(N-1)s}$ ,  $\theta_{Nc}$ ,  $\theta_{Ns}$ ,  $\theta_{(N+1)c}$ ,  $\theta_{(N+1)s}$

where

- $\theta$  is a Fourier Coefficient
- $N$  C is the number of blades
- $c$  refers to a cosine term
- $s$  refers to a sine term

consider a pair of monofrequency coefficients  $\theta_c$ ,  $\theta_s$

Let

$$y = \theta_s \sin \gamma + \theta_c \cos \gamma$$

where

- $\gamma$  is either  $(N-1)\Omega t$ , or  $N\Omega t$ , or  $(N+1)\Omega t$
- $\Omega$  is the rotor rotation rate
- $t$  is the time

Noting that

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

Then

$$y = \sqrt{\theta_s^2 + \theta_c^2} \left\{ \left( \frac{\theta_s}{\sqrt{\theta_s^2 + \theta_c^2}} \right) \sin \gamma + \left( \frac{\theta_c}{\sqrt{\theta_s^2 + \theta_c^2}} \right) \cos \gamma \right\}$$

or

$$y = \sqrt{\theta_s^2 + \theta_c^2} \{ \cos \psi \sin \gamma + \sin \psi \cos \gamma \}$$

where  $\psi$  is phase angle

and

$$\cos \psi = \frac{\theta_s}{\sqrt{\theta_s^2 + \theta_c^2}}, \quad \sin \psi = \frac{\theta_c}{\sqrt{\theta_s^2 + \theta_c^2}}$$

Then

$$\psi = \arcsin \left[ \frac{\theta_c}{\sqrt{\theta_s^2 + \theta_c^2}} \right] \quad \text{or} \quad \psi = \tan^{-1} \left( \frac{\theta_c}{\theta_s} \right)$$

So

$$y = \sqrt{\theta_s^2 + \theta_c^2} \sin(\gamma \pm \psi)$$

The magnitude constraint is just a constraint on the amplitude; hence the constraints become:

$$\theta_{(N-1)s}^2 + \theta_{(N-1)c}^2 \leq A_{N-1}^2$$

$$\theta_{Ns}^2 + \theta_{Nc}^2 \leq A_N^2$$

$$\theta_{(N+1)s}^2 + \theta_{(N+1)c}^2 \leq A_{N+1}^2$$

## 2.2 Nonrotating System Fourier Coefficients

If given the Fourier Coefficients for an entity expressed in the Nonrotating System, say  $\theta_{COL_c}$ ,  $\theta_{COL_s}$ ,  $\theta_{LAT_c}$ ,  $\theta_{LAT_s}$ ,  $\theta_{LONG_c}$ ,  $\theta_{LAT_s}$

where

- $\theta$  either a collective, or a lateral cyclic, or a longitudinal cyclic coefficient
- $COL$  refers to a collective coefficient
- $LAT$  refers to a lateral cyclic coefficient
- $LONG$  refers to a longitudinal cyclic coefficient
- $c$  refers to a cosine term
- $s$  refers to a sine term

then for oscillations at  $m$  per rev (see ref. 2)

$$\theta = \theta_{COL} + \theta_{LAT} \cos \Omega t + \theta_{LONG} \sin \Omega t$$

where

$$\begin{aligned} \theta_{COL} &= \theta_{COL_0} + \theta_{COL_s} \sin m\Omega t + \theta_{COL_c} \cos m\Omega t \\ \theta_{LAT} &= \theta_{LAT_0} + \theta_{LAT_s} \sin m\Omega t + \theta_{LAT_c} \cos m\Omega t \\ \theta_{LONG} &= \theta_{LONG_0} + \theta_{LONG_s} \sin m\Omega t + \theta_{LONG_c} \cos m\Omega t \\ t &\text{ is the time} \end{aligned}$$

In a manner similar to that employed for the Rotating System Fourier Coefficients (see sec. 2.1),  $\theta_{COL}$ ,  $\theta_{LAT}$ , and  $\theta_{LONG}$  can be expressed

$$\begin{aligned} \theta_{COL} &= \theta_{COL_0} + \sqrt{\theta_{COL_s}^2 + \theta_{COL_c}^2} \{ \cos \psi_{COL} \sin m\Omega t + \sin \psi_{COL} \cos m\Omega t \} \\ \theta_{LAT} &= \theta_{LAT_0} + \sqrt{\theta_{LAT_s}^2 + \theta_{LAT_c}^2} \{ \cos \psi_{LAT} \sin m\Omega t + \sin \psi_{LAT} \cos m\Omega t \} \\ \theta_{LONG} &= \theta_{LONG_0} + \sqrt{\theta_{LONG_s}^2 + \theta_{LONG_c}^2} \{ \cos \psi_{LONG} \sin m\Omega t + \sin \psi_{LONG} \cos m\Omega t \} \end{aligned}$$

where

$\psi_{COL}$ ,  $\psi_{LAT}$ , and  $\psi_{LONG}$  are phase angles

and

$$\begin{aligned} \cos \psi_{COL} &= \frac{\theta_{COL_s}}{\sqrt{\theta_{COL_s}^2 + \theta_{COL_c}^2}}, \quad \sin \psi_{COL} = \frac{\theta_{COL_c}}{\sqrt{\theta_{COL_s}^2 + \theta_{COL_c}^2}} \\ \cos \psi_{LAT} &= \frac{\theta_{LAT_s}}{\sqrt{\theta_{LAT_s}^2 + \theta_{LAT_c}^2}}, \quad \sin \psi_{LAT} = \frac{\theta_{LAT_c}}{\sqrt{\theta_{LAT_s}^2 + \theta_{LAT_c}^2}} \\ \cos \psi_{LONG} &= \frac{\theta_{LONG_s}}{\sqrt{\theta_{LONG_s}^2 + \theta_{LONG_c}^2}}, \quad \sin \psi_{LONG} = \frac{\theta_{LONG_c}}{\sqrt{\theta_{LONG_s}^2 + \theta_{LONG_c}^2}} \end{aligned}$$

Then

$$\begin{aligned} \psi_{COL} &= \arcsin [\sin \psi_{COL}] \quad \text{or} \quad \psi_{COL} = \tan^{-1} \left( \frac{\theta_{COL_c}}{\theta_{COL_s}} \right) \\ \psi_{LAT} &= \arcsin [\sin \psi_{LAT}] \quad \text{or} \quad \psi_{LAT} = \tan^{-1} \left( \frac{\theta_{LAT_c}}{\theta_{LAT_s}} \right) \\ \psi_{LONG} &= \arcsin [\sin \psi_{LONG}] \quad \text{or} \quad \psi_{LONG} = \tan^{-1} \left( \frac{\theta_{LONG_c}}{\theta_{LONG_s}} \right) \end{aligned}$$

The magnitude constraints on collective, lateral cyclic, and longitudinal cyclic are just constraints on their respective amplitudes; specifically:

$$\begin{aligned}\theta_{COL_s}^2 + \theta_{COL_c}^2 &\leq A_{COL}^2 \\ \theta_{LAT_s}^2 + \theta_{LAT_c}^2 &\leq A_{LAT}^2 \\ \theta_{LONG_s}^2 + \theta_{LONG_c}^2 &\leq A_{LONG}^2\end{aligned}$$

## 2.3 Individual Actuator Stroke

The magnitude constraints are just constraints on the vertical excursion of the individual actuators; specifically:

$$\begin{aligned}z_1 &\leq A_1 \\ z_2 &\leq A_2 \\ z_3 &\leq A_3\end{aligned}$$

If the motion of the actuators is sinusoidal,  $z_1, z_2$ , and  $z_3$  are expressed:

$$\begin{aligned}z_1 &= z_{0_1} + C_1 \sin(N\Omega t + \phi_1) \\ z_2 &= z_{0_2} + C_2 \sin(N\Omega t + \phi_2) \\ z_3 &= z_{0_3} + C_3 \sin(N\Omega t + \phi_3)\end{aligned}$$

where

- $N$  is the number of blades
- $\Omega$  is the rotor rotation rate
- $t$  is the time
- $\phi_i$  is phase angle for the  $i$ -th actuator;  $i = 1, 2, 3$
- $z_0$  is the reference actuator length

The magnitude constraints for the individual actuators becomes

$$\begin{aligned}C_1 &\leq A_1 \\ C_2 &\leq A_2 \\ C_3 &\leq A_3\end{aligned}$$

## 2.4 Blade Pitch Expressed as Function of Blade Azimuth and Actuator Stroke

To obtain an expression for the blade pitch as a function of blade azimuth and actuator stroke, the motion of the swashplate expressed as a function of the individual actuator strokes is derived first (see subsection 2.4.1). Next, the vertical position of a general field point (i.e., the end of a pitch link) is

defined (see subsection 2.4.2). Then finally, the blade pitch angle is defined from the vertical position of the pitch link (see subsection 2.4.2.3).

### 2.4.1 Canonical Swashplate Plane Equation

A necessary condition for a general field point  $P(x, y, z)$  to lie on the swashplate plane (see fig. 1) is:

$$(\bar{R} - \bar{R}_1) \cdot [(\bar{R}_2 - \bar{R}_1) \times (\bar{R}_3 - \bar{R}_1)] = 0 \quad (1)$$

$$\begin{aligned} \bar{R} &= \overline{OP} = x\hat{i} + y\hat{j} + z\hat{k} \\ \bar{R}_1 &= \overline{OP}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \\ \bar{R}_2 &= \overline{OP}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \\ \bar{R}_3 &= \overline{OP}_3 = x_3\hat{i} + y_3\hat{j} + z_3\hat{k} \end{aligned}$$

Let  $\mathcal{D}$  denote the matrix defined by

$$\mathcal{D} = \begin{bmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{bmatrix} \quad (2)$$

The swashplate plane defined by equation (1) can be expressed

$$\text{Det}(\mathcal{D}) = \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad (3)$$

or still

$$C_x(x - x_1) + C_y(y - y_1) + C_z(z - z_1) = 0 \quad (4)$$

where

$$\begin{aligned} C_x &= \text{cof}_{\mathcal{D}}(x - x_1) = (y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1) \\ C_y &= \text{cof}_{\mathcal{D}}(y - y_1) = (x_3 - x_1)(z_2 - z_1) - (x_2 - x_1)(z_3 - z_1) \\ C_z &= \text{cof}_{\mathcal{D}}(z - z_1) = (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \end{aligned}$$

Equation (4) can be rewritten as:

$$C_x x + C_y y + C_z z = C_x x_1 + C_y y_1 + C_z z_1$$

Let

$$D = C_x x_1 + C_y y_1 + C_z z_1$$

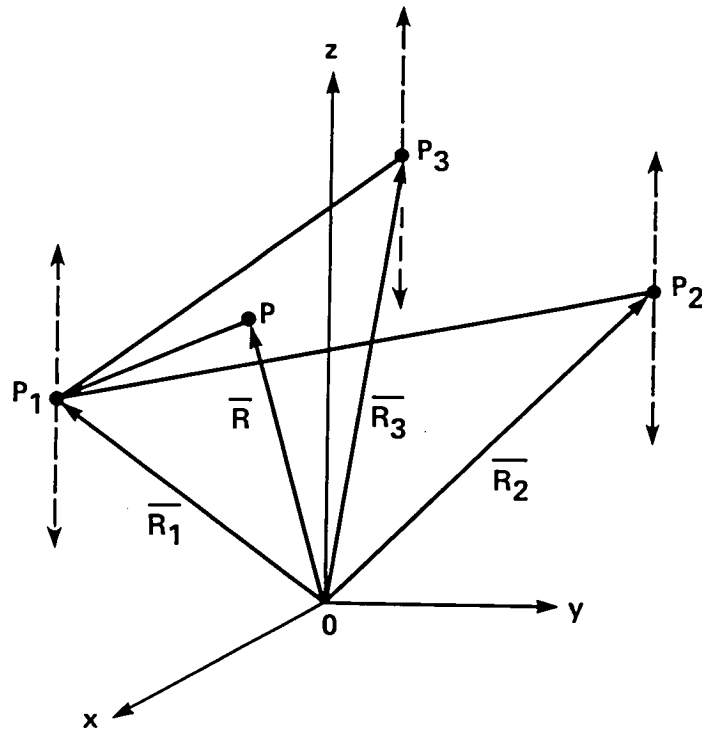
and

$$D_x = \frac{D}{C_x}, \quad D_y = \frac{D}{C_y}, \quad D_z = \frac{D}{C_z}$$

Then

$$\frac{x}{D_x} + \frac{y}{D_y} + \frac{z}{D_z} = 1 \quad \begin{array}{l} \text{Canonical form} \\ \text{of the swashplate} \\ \text{plane equation} \end{array} \quad (5)$$





Points  $P_1(x_1, y_1, z_1)$ ,  $P_2(x_2, y_2, z_2)$ , and  $P_3(x_3, y_3, z_3)$  define the swashplate plane.

Coordinates  $z_1, z_2, z_3$  define the vertical position of the actuator reference points on the swashplate plane.

The x-y plane is the Nonrotating System datum plane.

The z-axis is the Shaft Axis.

Figure 1. Swashplate plane geometry.

To evaluate  $D$ , note that

$$D = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Let  $\mathcal{R}$  denote the matrix defined by

$$\mathcal{R} = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

Then

$$D = \text{Det}(\mathcal{R}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \quad (6)$$

#### 2.4.2 Vertical Position of a General Field Point

The vertical position of a general field point on the swashplate plane is defined by solving equation (5) for  $z$ ; specifically:

$$z = D_z \left( 1 - \frac{x}{D_x} - \frac{y}{D_y} \right) \quad (7)$$

Recalling that

$$D_x = \frac{D}{C_x}, \quad D_y = \frac{D}{C_y}, \quad D_z = \frac{D}{C_z}$$

Equation (7) can be expressed

$$z = \frac{1}{C_z} (D - C_x x - C_y y) \quad (8)$$

Recalling that

$$\begin{aligned} C_x &= (y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1) \\ C_y &= (x_3 - x_1)(z_2 - z_1) - (x_2 - x_1)(z_3 - z_1) \\ C_z &= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \end{aligned}$$

and

$$D = \text{Det}(\mathcal{R}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Then

$$C_x = (y_3 - y_2)z_1 + (y_1 - y_3)z_2 + (y_2 - y_1)z_3 \quad (9)$$

$$C_y = (x_2 - x_3)z_1 + (x_3 - x_1)z_2 + (x_1 - x_2)z_3 \quad (10)$$

$$C_z = (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \quad (11)$$

and

$$D = (x_2y_3 - x_3y_2)z_1 + (x_3y_1 - x_1y_3)z_2 + (x_1y_2 - x_2y_1)z_3 \quad (12)$$

Note that  $z_1$ ,  $z_2$ , and  $z_3$  do not appear in equation (11).

Equation (8) can now be expressed

$$\begin{aligned} z = & [x_2y_3 - x_3y_2 - (y_3 - y_2)x - (x_2 - x_3)y]z_1 \\ & + [x_3y_1 - y_1y_3 - (y_1 - y_3)x - (x_3 - x_1)y]z_2 \\ & + [x_1y_2 - x_2y_1 - (y_2 - y_1)x - (x_1 - x_2)y]z_3 \\ & \frac{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)}{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)} \end{aligned} \quad (13)$$

which has the form

$$z = (A_1x + B_1y + C_1)z_1 + (A_2x + B_2y + C_2)z_2 + (A_3x + B_3y + C_3)z_3 \quad (14)$$

where

$$\left. \begin{aligned} A_1 &= \frac{-(y_3 - y_2)}{C_z} \\ B_1 &= \frac{-(x_2 - x_3)}{C_z} \\ C_1 &= \frac{x_2y_3 - x_3y_2}{C_z} \\ A_2 &= \frac{-(y_1 - y_3)}{C_z} \\ B_2 &= \frac{-(x_3 - x_1)}{C_z} \\ C_2 &= \frac{x_3y_1 - x_1y_3}{C_z} \\ A_3 &= \frac{-(y_2 - y_1)}{C_z} \\ B_3 &= \frac{-(x_1 - x_2)}{C_z} \\ C_3 &= \frac{x_1y_2 - x_2y_1}{C_z} \end{aligned} \right\} \text{Note: No } z\text{'s anywhere} \quad (15)$$

**2.4.2.1 Vertical position of a general field point expressed in the nonrotating system.** Equation (14), which is expressed in the Nonrotating System, defines the vertical position of a general field point on the swashplate plane. If the actuators move sinusoidally with  $N/\text{rev}$  frequency, the vertical position of the actuator reference points (i.e.,  $z_1$ ,  $z_2$ , and  $z_3$ ) can be expressed:

$$\begin{aligned} z_1 &= z_{01} + E_1 \sin(N\Omega t + \phi_1) \\ z_2 &= z_{02} + E_2 \sin(N\Omega t + \phi_2) \\ z_3 &= z_{03} + E_3 \sin(N\Omega t + \phi_3) \end{aligned} \quad (16)$$

Substitution of equations (16) into equation (14) yields:

$$\begin{aligned} z = & (A_1x + B_1y + C_1)[z_{01} + E_1 \sin(N\Omega t + \phi_1)] \\ & + (A_2x + B_2y + C_2)[z_{02} + E_2 \sin(N\Omega t + \phi_2)] \\ & + (A_3x + B_3y + C_3)[z_{03} + E_3 \sin(N\Omega t + \phi_3)] \end{aligned} \quad (17)$$

An appropriate Control 6-Vector is suggested; specifically:

$$U = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \quad (18)$$

Equation (17) with its three sinusoidal terms can be expressed in a form which has only one sinusoidal term by using the Rule for the Linear Combination of Monofrequency Sinusoidal Terms which is described in appendix A.

**2.4.2.2 Vertical position of a general field point expressed in the rotating system.** Equation (14), which is expressed in the Nonrotating System, can be applied to the Rotating System by defining the  $x$  and  $y$  coordinates of reference points on the pitch links as they rotate about the rotor shaft axis. For convenience in demonstrating the harmonic nature of the kinematics, it is assumed that the pitch links describe a right circular cylinder as they rotate about the rotor shaft axis (see fig. 2). It is emphasized that, in general, this assumption need not be made and that the pitch link motion need not describe a right circular cylinder in order to obtain the harmonic kinematic form derived subsequently. For those cases in which the up/down motion of the pitch links can be adequately described using small angle approximations, the  $x$  and  $y$  coordinates of the pitch links are expressed by:

$$\begin{aligned} x &= r \cos(\Omega t + \phi_s) \\ y &= r \sin(\Omega t + \phi_s) \end{aligned} \quad (19)$$

where

- $r$  is the radius of the right circular cylinder
- $t$  is the time
- $\Omega$  is the rotor rotation rate
- $\phi_s$  is the phase angle

If the actuators move sinusoidally with  $N/\text{rev}$  frequency, the vertical position of the actuator reference points (i.e.,  $z_1$ ,  $z_2$ , and  $z_3$ ) can be expressed:

$$\begin{aligned} z_1 &= z_{01} + E_1 \sin(N\Omega t + \phi_1) \\ z_2 &= z_{02} + E_2 \sin(N\Omega t + \phi_2) \\ z_3 &= z_{03} + E_3 \sin(N\Omega t + \phi_3) \end{aligned} \quad (20)$$

Substitution of equations (19) and (20) into equation (14) yields:

$$\begin{aligned} z &= [A_1 r \cos(\Omega t + \phi_s) + B_1 r \sin(\Omega t + \phi_s) + C_1][z_{01} + E_1 \sin(N\Omega t + \phi_1)] \\ &\quad + [A_2 r \cos(\Omega t + \phi_s) + B_2 r \sin(\Omega t + \phi_s) + C_2][z_{02} + E_2 \sin(N\Omega t + \phi_2)] \\ &\quad + [A_3 r \cos(\Omega t + \phi_s) + B_3 r \sin(\Omega t + \phi_s) + C_3][z_{03} + E_3 \sin(N\Omega t + \phi_3)] \end{aligned} \quad (21)$$

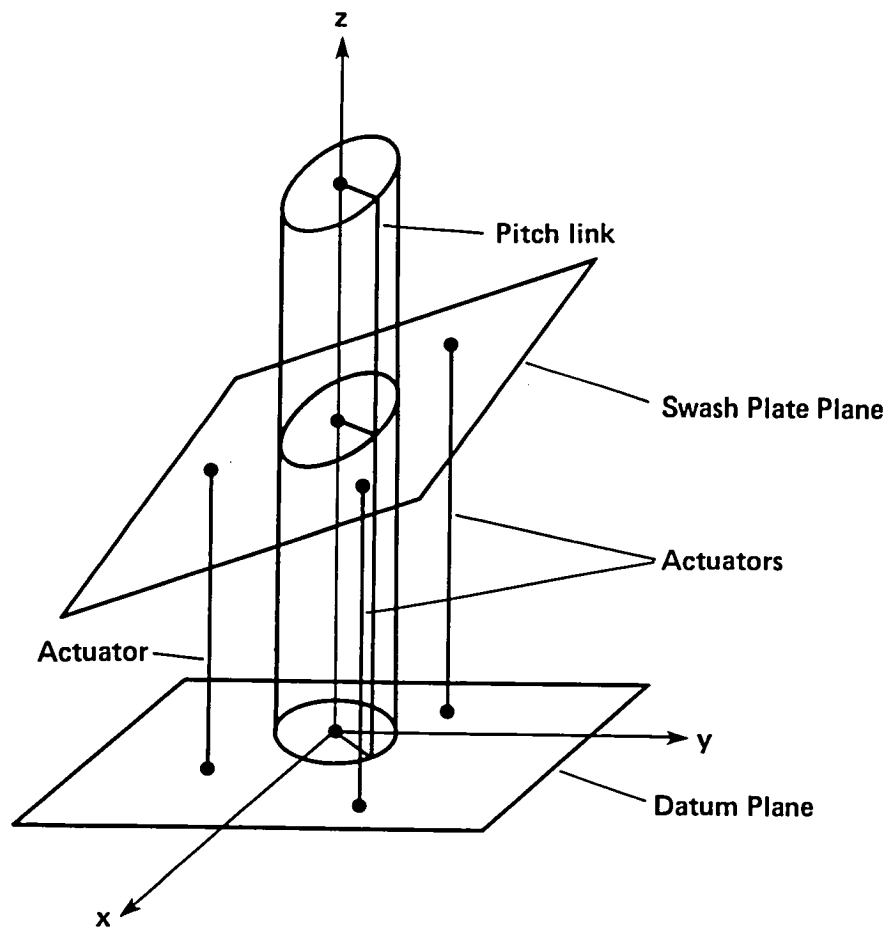


Figure 2. Actuator, swashplate plane, and pitch-link geometry.

As in the case of the Nonrotating System (see sec. 2.4.2.1), an appropriate Control 6-Vector is:

$$U = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \quad (22)$$

Expanding equation (21) by taking the indicated products yields:

$$\left. \begin{aligned} z = & A_1 z_{01} r \cos(\Omega t + \phi_s) + B_1 z_{01} r \sin(\Omega t + \phi_s) + C_1 z_{01} \\ & + A_1 E_1 r \sin(N\Omega t + \phi_1) \cos(\Omega t + \phi_s) + B_1 E_1 r \sin(N\Omega t + \phi_1) \sin(\Omega t + \phi_s) \\ & + C_1 E_1 \sin(N\Omega t + \phi_1) \\ & + A_2 z_{02} r \cos(\Omega t + \phi_s) + B_2 z_{02} r \sin(\Omega t + \phi_s) + C_2 z_{02} \\ & + A_2 E_2 r \sin(N\Omega t + \phi_2) \cos(\Omega t + \phi_s) + B_2 E_2 r \sin(N\Omega t + \phi_2) \sin(\Omega t + \phi_s) \\ & + C_2 E_2 \sin(N\Omega t + \phi_2) \\ & + A_3 z_{03} r \cos(\Omega t + \phi_s) + B_3 z_{03} r \sin(\Omega t + \phi_s) + C_3 z_{03} \\ & + A_3 E_3 r \sin(N\Omega t + \phi_3) \cos(\Omega t + \phi_s) + B_3 E_3 r \sin(N\Omega t + \phi_3) \sin(\Omega t + \phi_s) \\ & + C_3 E_3 \sin(N\Omega t + \phi_3) \end{aligned} \right\} \quad (23)$$

Noting that:

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \sin \alpha \sin \beta &= -\frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta) \\ \sin \alpha \cos \beta &= \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta) \\ \cos \alpha \sin \beta &= \frac{1}{2} \sin(\alpha + \beta) - \frac{1}{2} \sin(\alpha - \beta) \\ \cos \alpha \cos \beta &= \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta) \end{aligned}$$

Equation (23) is further expanded

$$\begin{aligned}
z = & A_1 z_{01} r [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] + B_1 z_{01} r [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
& + C_1 z_{01} + A_1 E_1 r [\cos \phi_1 \sin N\Omega t + \sin \phi_1 \cos N\Omega t] [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] \\
& + B_1 E_1 r [\cos \phi_1 \sin N\Omega t + \sin \phi_1 \cos N\Omega t] [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
& + C_1 E_1 [\cos \phi_1 \sin N\Omega t + \sin \phi_1 \cos N\Omega t] \\
& + A_2 z_{02} r [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] + B_2 z_{02} r [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
& + C_2 z_{02} + A_2 E_2 r [\cos \phi_2 \sin N\Omega t + \sin \phi_2 \cos N\Omega t] [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] \\
& + B_2 E_2 r [\cos \phi_2 \sin N\Omega t + \sin \phi_2 \cos N\Omega t] [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
& + C_2 E_2 [\cos \phi_2 \sin N\Omega t + \sin \phi_2 \cos N\Omega t] \\
& + A_3 z_{03} r [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] + B_3 z_{03} r [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
& + C_3 z_{03} + A_3 E_3 r [\cos \phi_3 \sin N\Omega t + \sin \phi_3 \cos N\Omega t] [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] \\
& + B_3 E_3 r [\cos \phi_3 \sin N\Omega t + \sin \phi_3 \cos N\Omega t] [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
& + C_3 E_3 [\cos \phi_3 \sin N\Omega t + \sin \phi_3 \cos N\Omega t]
\end{aligned} \tag{24}$$

Expanding the products for  $k = 1, 2, 3$

$$\begin{aligned}
& [\cos \phi_k \sin N\Omega t + \sin \phi_k \cos N\Omega t] [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] \\
& = \cos \phi_k \cos \phi_s \sin N\Omega t \cos \Omega t + \sin \phi_k \cos \phi_s \cos N\Omega t \cos \Omega t \\
& \quad - \cos \phi_k \sin \phi_s \sin N\Omega t \sin \Omega t - \sin \phi_k \sin \phi_s \cos N\Omega t \sin \Omega t \\
& = \frac{1}{2} \cos \phi_k \cos \phi_s \{ \sin[(N+1)\Omega t] + \sin[(N-1)\Omega t] \} \\
& \quad + \frac{1}{2} \sin \phi_k \cos \phi_s \{ \cos[(N+1)\Omega t] + \cos[(N-1)\Omega t] \} \\
& \quad + \frac{1}{2} \cos \phi_k \sin \phi_s \{ \cos[(N+1)\Omega t] - \cos[(N-1)\Omega t] \} \\
& \quad - \frac{1}{2} \sin \phi_k \sin \phi_s \{ \sin[(N+1)\Omega t] - \sin[(N-1)\Omega t] \} \\
& = \frac{1}{2} [\cos \phi_k \cos \phi_s - \sin \phi_k \sin \phi_s] \sin[(N+1)\Omega t] \\
& \quad + \frac{1}{2} [\sin \phi_k \cos \phi_s + \cos \phi_k \sin \phi_s] \cos[(N+1)\Omega t] \\
& \quad + \frac{1}{2} [\cos \phi_k \cos \phi_s + \sin \phi_k \sin \phi_s] \sin[(N-1)\Omega t] \\
& \quad + \frac{1}{2} [\sin \phi_k \cos \phi_s - \cos \phi_k \sin \phi_s] \cos[(N-1)\Omega t] \\
& = \frac{1}{2} \cos(\phi_k + \phi_s) \sin[(N+1)\Omega t] \\
& \quad + \frac{1}{2} \sin(\phi_k + \phi_s) \cos[(N+1)\Omega t] \\
& \quad + \frac{1}{2} \cos(\phi_k - \phi_s) \sin[(N-1)\Omega t] \\
& \quad + \frac{1}{2} \sin(\phi_k - \phi_s) \cos[(N-1)\Omega t]
\end{aligned} \tag{25}$$

for  $k = 1, 2, 3$

and likewise:

$$\begin{aligned}
& [\cos \phi_k \sin N\Omega t + \sin \phi_k \cos N\Omega t][\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
&= \cos \phi_k \cos \phi_s \sin N\Omega t \sin \Omega t + \sin \phi_k \cos \phi_s \cos N\Omega t \sin \Omega t \\
&\quad + \cos \phi_k \sin \phi_s \sin N\Omega t \cos \Omega t + \sin \phi_k \sin \phi_s \cos N\Omega t \cos \Omega t \\
&= \frac{1}{2} \cos \phi_k \cos \phi_s \{-\cos[(N+1)\Omega t] + \cos[(N-1)\Omega t]\} \\
&\quad + \frac{1}{2} \sin \phi_k \cos \phi_s \{\sin[(N+1)\Omega t] - \sin[(N-1)\Omega t]\} \\
&\quad + \frac{1}{2} \cos \phi_k \sin \phi_s \{\sin[(N+1)\Omega t] + \sin[(N-1)\Omega t]\} \\
&\quad + \frac{1}{2} \sin \phi_k \sin \phi_s \{\cos[(N+1)\Omega t] + \cos[(N-1)\Omega t]\} \\
&= \frac{1}{2} [\sin \phi_k \cos \phi_s + \cos \phi_k \sin \phi_s] \sin[(N+1)\Omega t] \\
&\quad - \frac{1}{2} [\cos \phi_k \cos \phi_s - \sin \phi_k \sin \phi_s] \cos[(N+1)\Omega t] \\
&\quad - \frac{1}{2} [\sin \phi_k \cos \phi_s - \cos \phi_k \sin \phi_s] \sin[(N-1)\Omega t] \\
&\quad + \frac{1}{2} [\cos \phi_k \cos \phi_s + \sin \phi_k \sin \phi_s] \cos[(N-1)\Omega t] \\
&= \frac{1}{2} \sin(\phi_k + \phi_s) \sin[(N+1)\Omega t] \\
&\quad - \frac{1}{2} \cos(\phi_k + \phi_s) \cos[(N+1)\Omega t] \\
&\quad - \frac{1}{2} \sin(\phi_k - \phi_s) \sin[(N-1)\Omega t] \\
&\quad + \frac{1}{2} \cos(\phi_k - \phi_s) \cos[(N-1)\Omega t]
\end{aligned}
\tag{26}$$

for  $k = 1, 2, 3$



Substituting equations (25) and (26) into equation (24) yields

$$\begin{aligned}
 z = & A_1 z_{01} r [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] + B_1 z_{01} r [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
 & + C_1 z_{01} + \frac{1}{2} A_1 E_1 r \{ \cos(\phi_1 + \phi_s) \sin[(N+1)\Omega t] + \sin(\phi_1 + \phi_s) \cos[(N+1)\Omega t] \\
 & + \cos(\phi_1 - \phi_s) \sin[(N-1)\Omega t] + \sin(\phi_1 - \phi_s) \cos[(N-1)\Omega t] \} \\
 & + \frac{1}{2} B_1 E_1 r \{ \sin(\phi_1 + \phi_s) \sin[(N+1)\Omega t] - \cos(\phi_1 + \phi_s) \cos[(N+1)\Omega t] \\
 & - \sin(\phi_1 - \phi_s) \sin[(N-1)\Omega t] + \cos(\phi_1 - \phi_s) \cos[(N-1)\Omega t] \} \\
 & + C_1 E_1 [\cos \phi_1 \sin N\Omega t + \sin \phi_1 \cos N\Omega t] \\
 & + A_2 z_{02} r [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] + B_2 z_{02} r [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
 & + C_2 z_{02} + \frac{1}{2} A_2 E_2 r \{ \cos(\phi_2 + \phi_s) \sin[(N+1)\Omega t] + \sin(\phi_2 + \phi_s) \cos[(N+1)\Omega t] \\
 & + \cos(\phi_2 - \phi_s) \sin[(N-1)\Omega t] + \sin(\phi_2 - \phi_s) \cos[(N-1)\Omega t] \} \\
 & + \frac{1}{2} B_2 E_2 r \{ \sin(\phi_2 + \phi_s) \sin[(N+1)\Omega t] - \cos(\phi_2 + \phi_s) \cos[(N+1)\Omega t] \\
 & - \sin(\phi_2 - \phi_s) \sin[(N-1)\Omega t] + \cos(\phi_2 - \phi_s) \cos[(N-1)\Omega t] \} \\
 & + C_2 E_2 [\cos \phi_2 \sin N\Omega t + \sin \phi_2 \cos N\Omega t] \\
 & + A_3 z_{03} r [\cos \phi_s \cos \Omega t - \sin \phi_s \sin \Omega t] + B_3 z_{03} r [\cos \phi_s \sin \Omega t + \sin \phi_s \cos \Omega t] \\
 & + C_3 z_{03} + \frac{1}{2} A_3 E_3 r \{ \cos(\phi_3 + \phi_s) \sin[(N+1)\Omega t] + \sin(\phi_3 + \phi_s) \cos[(N+1)\Omega t] \\
 & + \cos(\phi_3 - \phi_s) \sin[(N-1)\Omega t] + \sin(\phi_3 - \phi_s) \cos[(N-1)\Omega t] \} \\
 & + \frac{1}{2} B_3 E_3 r \{ \sin(\phi_3 + \phi_s) \sin[(N+1)\Omega t] - \cos(\phi_3 + \phi_s) \cos[(N+1)\Omega t] \\
 & - \sin(\phi_3 - \phi_s) \sin[(N-1)\Omega t] + \cos(\phi_3 - \phi_s) \cos[(N-1)\Omega t] \} \\
 & + C_3 E_3 [\cos \phi_3 \sin N\Omega t + \sin \phi_3 \cos N\Omega t]
 \end{aligned} \tag{27}$$

Rearranging the terms in equation (27) results in:

$$\begin{aligned}
 z = & [C_1 z_{01} + C_2 z_{02} + C_3 z_{03}] \\
 & + r[-A_1 z_{01} \sin \phi_s + B_1 z_{01} \cos \phi_s - A_2 z_{02} \sin \phi_s \\
 & + B_2 z_{02} \cos \phi_s - A_3 z_{03} \sin \phi_s + B_3 z_{03} \cos \phi_s] \sin \Omega t \\
 & + r[A_1 z_{01} \cos \phi_s + B_1 z_{01} \sin \phi_s + A_2 z_{02} \cos \phi_s \\
 & + B_2 z_{02} \sin \phi_s + A_3 z_{03} \cos \phi_s + B_3 z_{03} \sin \phi_s] \cos \Omega t \\
 & + \frac{1}{2} r[A_1 E_1 \cos(\phi_1 - \phi_s) - B_1 E_1 \sin(\phi_1 - \phi_s) + A_2 E_2 \cos(\phi_2 - \phi_s) \\
 & - B_2 E_2 \sin(\phi_2 - \phi_s) + A_3 E_3 \cos(\phi_3 - \phi_s) - B_3 E_3 \sin(\phi_3 - \phi_s)] \sin[(N-1)\Omega t] \\
 & + \frac{1}{2} r[A_1 E_1 \sin(\phi_1 - \phi_s) - B_1 E_1 \cos(\phi_1 - \phi_s) + A_2 E_2 \sin(\phi_2 - \phi_s) \\
 & + B_2 E_2 \cos(\phi_2 - \phi_s) + A_3 E_3 \sin(\phi_3 - \phi_s) - B_3 E_3 \cos(\phi_3 - \phi_s)] \cos[(N-1)\Omega t] \\
 & + [C_1 E_1 \cos \phi_1 + C_2 E_2 \cos \phi_2 + C_3 E_3 \cos \phi_3] \sin N\Omega t \\
 & + [C_1 E_1 \sin \phi_1 + C_2 E_2 \sin \phi_2 + C_3 E_3 \sin \phi_3] \cos N\Omega t \\
 & + \frac{1}{2} r[A_1 E_1 \cos(\phi_1 + \phi_s) + B_1 E_1 \sin(\phi_1 + \phi_s) + A_2 E_2 \cos(\phi_2 + \phi_s) \\
 & + B_2 E_2 \sin(\phi_2 + \phi_s) + A_3 E_3 \cos(\phi_3 + \phi_s) + B_3 E_3 \sin(\phi_3 + \phi_s)] \sin[(N+1)\Omega t] \\
 & + \frac{1}{2} r[A_1 E_1 \sin(\phi_1 + \phi_s) - B_1 E_1 \cos(\phi_1 + \phi_s) + A_2 E_2 \sin(\phi_2 + \phi_s) \\
 & - B_2 E_2 \cos(\phi_2 + \phi_s) + A_3 E_3 \sin(\phi_3 + \phi_s) - B_3 E_3 \cos(\phi_3 + \phi_s)] \sin[(N+1)\Omega t]
 \end{aligned} \tag{28}$$

Note:  $z$  represents an arbitrary vertical reference position along a pitch link

**2.4.2.3 Blade pitch.** A simplified model of the pitch link, pitch link connecting arm, and blade angle geometry is presented below to illustrate the nature of the geometry. This geometry is rotor head peculiar and, correspondingly, each case requires individual analysis.

It is assumed that the pitch link is parallel to the rotor shaft about which it rotates and that it describes a right circular cylinder as it rotates about the rotor shaft (see fig. 2). It is further assumed that the pitch link connecting arm is nearly perpendicular to the pitch link and to the blade pitch axis (see fig. 3). Finally, it is assumed that the pitch angle  $\theta$  is small enough so that

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1$$

Then

$$l(\theta + \theta_0) = z - z_{00} \tag{29}$$

or

$$\theta = \frac{z - z_{00}}{l} - \theta_0$$

where

$\theta$  is the incremental blade pitch angle measured from  $\theta_0$

$\theta_0$  is an arbitrary baseline trim value of  $\theta$

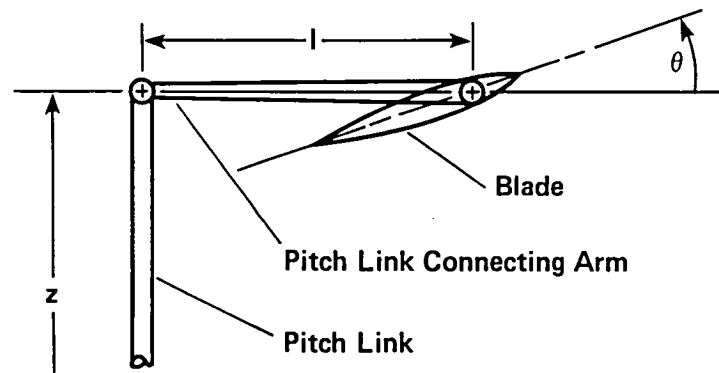


Figure 3. Pitch link, pitch-link connecting arm, and blade-angle geometry.

- $l$  is the pitch link connecting arm length
- $z$  is the vertical position of the end of the pitch link
- $z_{00}$  is the reference value of  $z$  which corresponds to  $\theta_0$

## 2.5 Rate Control

Rate control is achieved by first taking the time derivative of the entity whose rate is to be constrained, and then by defining a constraint function for this time derivative. The entities whose rates are to be determined have one of two basic forms; these are:

$$y = A \sin \omega t + B \cos \omega t$$

and

$$y = y_0 + C \sin(\omega t + \gamma)$$

The first form is used for Fourier equations expressed in both the Rotating and Nonrotating Systems as described in sections 2.1 and 2.2, respectively. The second form is used for the sinusoidal individual actuator strokes described in section 2.3. The time derivative and constraint function for each of these forms are presented in subsections 2.5.1 and 2.5.2, respectively.

### 2.5.1 Rates for the Form $y = A \sin \omega t + B \cos \omega t$

If given

$$y = A \sin \omega t + B \cos \omega t \tag{30}$$

Then

$$\dot{y} = \omega A \cos \omega t - \omega B \sin \omega t$$

Noting that

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\dot{y} = \sqrt{(\omega A)^2 + (\omega B)^2} \left\{ \left( \frac{\omega A}{\sqrt{(\omega A)^2 + (\omega B)^2}} \right) \cos \omega t - \left( \frac{\omega B}{\sqrt{(\omega A)^2 + (\omega B)^2}} \right) \sin \omega t \right\}$$

$$\dot{y} = \omega \sqrt{A^2 + B^2} \{ \cos \omega t \cos \gamma - \sin \omega t \sin \gamma \}$$

where  $\gamma$  is the phase angle defined by

$$\cos \gamma = \frac{A}{\sqrt{A^2 + B^2}}, \quad \sin \gamma = \frac{B}{\sqrt{A^2 + B^2}}$$

$$\gamma = \arccos(\cos \gamma) \quad \text{or} \quad \gamma = \tan^{-1} \left( \frac{B}{A} \right)$$

So:

$$\dot{y} = \omega \sqrt{A^2 + B^2} \cos(\omega t + \gamma) \quad (31)$$

The rate constraint is just a constraint on the amplitude of equation (31); specifically:

$$\omega \sqrt{A^2 + B^2} \leq A_R \quad (32)$$

### 2.5.2 Rates for the Form $y = y_0 + C \sin(\omega t + \gamma)$

If given

$$y = y_0 + C \sin(\omega t + \gamma) \quad (33)$$

Then

$$\dot{y} = \omega C \cos(\omega t + \gamma) \quad (34)$$

The rate constraint is just a constraint on the amplitude of equation (5); specifically:

$$\omega C \leq A_R \quad (35)$$

## 2.6 Power

The hydraulic power required to operate the actuators can be the limiting factor in the use of Higher Harmonic Control to reduce helicopter vibration. Correspondingly, it is desirable to be able to define a constraint function which limits the hydraulic power and which is expressed in terms of variables which are computed during the rotor simulation. Accordingly, the following procedure was defined to express hydraulic power in terms of mechanical variables.

The Hydraulic Power ( $P_H$ ) requirement is roughly proportional to the Mechanical Power ( $P_M$ ) generated; specifically:

$$P_H = K_p P_M \quad (36)$$

where  $K_p$  is the constant of proportionality.

The Mechanical Power ( $P_M$ ) is proportional to the time rate-of-change of the Total Mechanical Energy (E) of the system; specifically:

$$P_M = K_E \frac{d}{dt}(E) \quad (37)$$

where  $K_E$  is the constant of proportionality.

For the systems under consideration, the Total Mechanical Energy (E) can be assumed to be comprised of the Kinetic Energy (T) and the Potential Energy (V) due to the presence of an effective spring constant, and is the sum of the individual mechanical energies of several subsystems. These subsystems include nonrotating system components such as the actuators themselves and rotating system components such as the individual blades with hinges and linkages. The Total Mechanical Energy for each of these subsystems is:

$$E = T + V \quad (38)$$

The Kinetic Energy (T) is proportional to  $(dz/dt)^2$  in  $ml^2/t^2$ .

The Potential Energy (V) for an effective spring constant  $K$  is:

$$V = V_2 - V_1 = \int_{z_{00}}^z F_s d\xi \quad (39)$$

where

$$F_s = K(\xi - z_0)$$

$K$  the effective spring constant with units of  $\frac{F}{l} = \frac{m}{t^2}$

$z_0$  vertical displacement at which the effective spring force is zero

$z_{00}$  reference vertical displacement at which  $V_1$  is defined

Then

$$V = K \int_{z_{00}}^z (\xi - z_0) d\xi = K \left[ \frac{1}{2} \xi^2 - z_0 \xi \right]_{z_{00}}^z \quad (40)$$

$$V = K \left[ \frac{1}{2} z^2 - z_0 z + \left( z_0 z_{00} - \frac{1}{2} z_{00}^2 \right) \right] \text{ in } \frac{ml^2}{t^2} \quad (41)$$

which can be expressed as

$$V = Fz^2 + Gz + H \text{ in } \frac{ml^2}{t^2} \quad (42)$$

where:

$$F = \frac{1}{2}K, \quad G = -z_0K, \quad H = K \left( z_0 z_{00} - \frac{1}{2} z_{00}^2 \right)$$

Then

$$E = D \left( \frac{dz}{dt} \right)^2 + Fz^2 + Gz + H \text{ in } \frac{ml^2}{t^2} \quad (43)$$

where  $D$  is the constant of proportionality for  $T$

Recalling equation (31), the Mechanical Power ( $P_M$ ) can be expressed as:

$$P_M = K_E \frac{d}{df} \left[ D \left( \frac{dz}{dt} \right)^2 + Fz^2 + Gz + H \right] \text{ in } \frac{ml^2}{t^3} \quad (44)$$

$$P_M = K_E \left[ 2D \left( \frac{dz}{dt} \right) \left( \frac{d^2z}{dt^2} \right) + 2Fz \left( \frac{dz}{dt} \right) + G \left( \frac{dz}{dt} \right) \right] \quad (45)$$

$$P_M = K_E \left[ 2D \left( \frac{d^2z}{dt^2} \right) + 2Fz + G \right] \left( \frac{dz}{dt} \right) \text{ in } \frac{ml^2}{t^3} \quad (46)$$

### 2.6.1 Nonrotating Subsystems

It is assumed that the three actuators with their associated moving linkages are the three principal nonrotating subsystems which must be considered. The Mechanical Power  $P_M$  for each of the actuator subsystems is expressed by equation (46) using the appropriate  $z$  position defined by equation (16) in section 2.4.2.1; specifically if

$$\left. \begin{aligned} z_1 &= z_{01} + E_1 \sin(N\Omega t + \phi_1) \\ z_2 &= z_{02} + E_2 \sin(N\Omega t + \phi_2) \\ z_3 &= z_{03} + E_3 \sin(N\Omega t + \phi_3) \end{aligned} \right\} \text{Equation (16) in section 2.4.2.1}$$

then for actuator  $k, k = 1, 2, 3$  the Mechanical Power  $P_M$  is:

$$\begin{aligned} P_{M_k} &= N\Omega E_k K_E [2D\{-N^2\Omega^2 E_k \sin(N\Omega t + \phi_k)\} \\ &\quad + 2F\{z_{0_k} + E_k \sin(N\Omega t + \phi_k)\} + G] \cos(N\Omega t + d_k) \end{aligned} \quad (47)$$

for  $k = 1, 2, 3$

Let  $sc(N\Omega t)$  denote either  $\sin(N\Omega t)$  or  $\cos(N\Omega t)$  and noting that

$$\begin{aligned} \sin(N\Omega t + \phi_k) &= \cos \phi_k \sin(N\Omega t) + \sin \phi_k \cos(N\Omega t) \\ \cos(N\Omega t + \phi_k) &= \cos \phi_k \cos(N\Omega t) - \sin \phi_k \sin(N\Omega t) \\ \sin^2(N\Omega t) &= \frac{1}{2}[1 - \cos(2N\Omega t)] \\ \sin(N\Omega t) \cos(N\Omega t) &= \frac{1}{2} \sin(2N\Omega t) \\ \cos^2(N\Omega t) &= \frac{1}{2}[1 + \cos(2N\Omega t)] \end{aligned}$$

Equation (47) for the  $k$ th actuator subsystem has the form

$$\begin{aligned} P_{M_k} &= B_1 \sin(N\Omega t) + B_2 \cos(N\Omega t) + B_3 \sin^2(N\Omega t) + B_4 \sin(N\Omega t) \cos(N\Omega t) \\ &\quad + B_5 \cos^2(N\Omega t) \end{aligned} \quad (48)$$

$$P_{M_k} = C_0 + C_1 \sin(N\Omega t + \zeta_1) + C_2 \sin(2N\Omega t + \zeta_2) \quad (49)$$

where  $B_1, B_2, B_3, B_4, B_5, C_0, C_1, C_2, \zeta_1$ , and  $\zeta_2$  are constants. The total Mechanical Power  $P_M$  for all three actuator subsystems is defined by summing the Mechanical Power  $P_{M_k}$  of each of these subsystems. The result has the same form as that of equation (49), but where the constants are different; specifically

$$P_M = D_0 + D_1 \sin(N\Omega t + \psi_1) + D_2 \sin(2N\Omega t + \psi_2) \quad (50)$$

where  $D_0, D_1, D_2, \psi_1$ , and  $\psi_2$  are constants.

If the power required by just the three nonrotating actuator subsystems is to be considered, then the power constraint can be expressed as:

$$|D_0| + |D_1| + |D_2| \leq [P_M]_{max} \quad (51)$$

### 2.6.2 Rotating Subsystems

It is assumed that the  $N$  individual blades with their hinges and linkages are the  $N$  principal rotating subsystems which must be considered. The Mechanical Power  $P_M$  for each of the individual blades is expressed by equation (46) using the  $z$  position defined by equation (28) in section 2.4.2.2 where the appropriate phase angle  $\phi_{s_i}$  for the  $i$ -th blade is:

$$\phi_{s_i} = (i - 1) \left( \frac{360^\circ}{N} \right) \quad \text{for } i = 1, 2, \dots, N \quad (52)$$

where

$N$     number of blades in the rotor  
 $i$     blade number;  $i = 1, 2, \dots, N$

In a manner similar to that employed for the nonrotating subsystems, the aforementioned procedure (i.e., substitution of eq. (28) of sec. 2.4.2.2 into eq. (46)) will yield the power requirement for an individual blade subsystem. Combining these results for each individual blade, an expression can be obtained for the power requirements for all the rotating subsystems. This expression is the product of two terms comprised of various harmonics of  $\Omega t$ ; specifically:

$$P_M = (\text{TERM}_1)(\text{TERM}_2) \quad (53)$$

where:

$$\begin{aligned} \text{TERM}_1 &= A_0 + A_1 \text{sc}(\Omega t) + A_2 \text{sc}[(N - 1)\Omega t] + A_3 \text{sc}(N\Omega t) + A_4 \text{sc}[(N + 1)\Omega t] \\ \text{TERM}_2 &= B_1 \text{sc}(\Omega t) + B_2 \text{sc}[(N - 1)\Omega t] + B_3 \text{sc}(N\Omega t) + B_4 \text{sc}[(N + 1)\Omega t] \end{aligned}$$

and

$$\text{sc}(\cdot) \text{ denotes either } \sin(\cdot) \text{ or } \cos(\cdot) \quad (54)$$

$P_M$  then has the form

$$\begin{aligned} P_M &= C_1 \text{sc}(\Omega t) + C_2 \text{sc}[(N - 1)\Omega t] + C_3 \text{sc}(N\Omega t) + C_4 \text{sc}[(N + 1)\Omega t] \\ &\quad + C_5 \text{sc}(\Omega t) \text{sc}(\Omega t) + C_6 \text{sc}[(N - 1)\Omega t] \text{sc}(\Omega t) \\ &\quad + C_7 \text{sc}(N\Omega t) \text{sc}(\Omega t) + C_8 \text{sc}[(N + 1)\Omega t] \text{sc}(\Omega t) \\ &\quad + C_9 \text{sc}[(N - 1)\Omega t] \text{sc}[(N - 1)\Omega t] + C_{10} \text{sc}(N\Omega t) \text{sc}[(N - 1)\Omega t] \\ &\quad + C_{11} \text{sc}[(N + 1)\Omega t] \text{sc}[(N - 1)\Omega t] \\ &\quad + C_{12} \text{sc}(N\Omega t) \text{sc}(N\Omega t) + C_{13} \text{sc}[(N + 1)\Omega t] \text{sc}(N\Omega t) \\ &\quad + C_{14} \text{sc}[(N + 1)\Omega t] \text{sc}[(N + 1)\Omega t] \end{aligned} \quad (55)$$

The following rules and identities are used to simplify equation (55)

$$\left. \begin{aligned} 1. \quad &\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ &\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{aligned} \right\} \quad (56)$$



$$2. \left. \begin{aligned} \sin \alpha \sin \beta &= -\frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta) \\ \sin \alpha \cos \beta &= \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta) \\ \cos \alpha \sin \beta &= \frac{1}{2} \sin(\alpha + \beta) - \frac{1}{2} \sin(\alpha - \beta) \\ \cos \alpha \cos \beta &= \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta) \end{aligned} \right\} \quad (57)$$

3. Rule for the Linear Combination of Monofrequency Sinusoidal Terms (see app. A).

Using the above defined rules and identities, equation (55) can be rearranged to a form with no sinusoidal products.

$$\begin{aligned} P_M &= D_0 + D_1 \sin[\Omega t + \sigma_1] + D_2 \sin[2(\Omega t) + \sigma_2] \\ &+ D_3 \sin[(N-2)\Omega t + \sigma_3] + D_4 \sin[(N-1)\Omega t + \sigma_4] \\ &+ D_5 \sin[N(\Omega t) + \sigma_5] + D_6 \sin[(N+1)\Omega t + \sigma_6] \\ &+ D_7 \sin[(N+2)\Omega t + \sigma_7] + D_8 \sin[(2N-2)\Omega t + \sigma_8] \\ &+ D_9 \sin[(2N-1)\Omega t + \sigma_9] + D_{10} \sin[2N(\Omega t) + \sigma_{10}] \\ &+ D_{11} \sin[(2N+1)\Omega t + \sigma_{11}] + D_{12} \sin[(2N+2)\Omega t + \sigma_{12}] \end{aligned} \quad (58)$$

where

Table 1.

Coefficient in equation (55)	Sinusoid in equation (55)	Yields sinusoidal terms of the form
$C_1$	$sc(\Omega t)$	$\Rightarrow sc(\Omega t)$
$C_2$	$sc[(N-1)\Omega t]$	$\Rightarrow sc[(N-1)\Omega t]$
$C_3$	$sc[N(\Omega t)]$	$\Rightarrow sc[N(\Omega t)]$
$C_4$	$sc[(N+1)\Omega t]$	$\Rightarrow sc[(N+1)\Omega t]$
$C_5$	$sc(\Omega t)sc(\Omega t)$	$\Rightarrow sc[2(\Omega t)]$ and a constant
$C_6$	$sc[(N-1)\Omega t]sc(\Omega t)$	$\Rightarrow sc[N(\Omega t)]$ and $sc[(N-2)\Omega t]$
$C_7$	$sc[N(\Omega t)]sc(\Omega t)$	$\Rightarrow sc[(N+1)\Omega t]$ and $sc[(N-1)\Omega t]$
$C_8$	$sc[(N+1)\Omega t]sc(\Omega t)$	$\Rightarrow sc[(N+2)\Omega t]$ and $sc[N(\Omega t)]$
$C_9$	$sc[(N-1)\Omega t]sc[(N-1)\Omega t]$	$\Rightarrow sc[(2N-2)\Omega t]$ and a constant
$C_{10}$	$sc[N(\Omega t)]sc[(N-1)\Omega t]$	$\Rightarrow sc[(2N-1)\Omega t]$ and $sc(\Omega t)$
$C_{11}$	$sc[(N+1)\Omega t]sc[(N-1)\Omega t]$	$\Rightarrow sc[2N(\Omega t)]$ and $sc[2(\Omega t)]$
$C_{12}$	$sc[N(\Omega t)]sc[N(\Omega t)]$	$\Rightarrow sc[2N(\Omega t)]$ and a constant
$C_{13}$	$sc[(N+1)\Omega t]sc[N(\Omega t)]$	$\Rightarrow sc[(2N+1)\Omega t]$ and $sc(\Omega t)$
$C_{14}$	$sc[(N+1)\Omega t]sc[(N+1)\Omega t]$	$\Rightarrow sc[(2N+2)\Omega t]$ and constant

The power constraint can be written as:

$$P_H = K_P P_M \leq [P_H]_{MAX} \quad (59)$$

or

$$K_P \sum_{j=0}^{12} |D_j| \leq [P_H]_{MAX}$$

### 3.0 RESULTS

Six constraint functions associated with swashplate blade pitch control were defined by expressing them in explicit equation form. These functions constrain: 1) blade pitch Fourier Coefficients expressed in the Rotating System, 2) blade pitch Fourier Coefficients expressed in the Nonrotating System, 3) stroke of the individual actuators expressed in the Nonrotating System, 4) blade pitch expressed as a function of blade azimuth and actuator stroke, 5) time rate-of-change of the aforementioned parameters, and 6) required power. In addition to the aforementioned constraints, the kinematics of swashplate blade pitch control referenced to the individual strokes of the actuators in the nonrotating system were derived using vector analysis.

### 4.0 REFERENCES

1. Leyland, Jane Anne: Comparison of Three Controllers Applied to Helicopter Vibration. NASA TM-102192, May 1992.
2. Biggers, James C.; and McCloud, John L., III: A Note on Multicyclic Control by Swashplate Oscillation. NASA TM-78475, April 1978.

# APPENDIX A

## RULE FOR THE LINEAR COMBINATION OF MONOFREQUENCY SINUSOIDAL TERMS

Consider first the sum of three monofrequency sinusoidal terms; specifically if given

$$w = F_1 \sin(\theta + \phi_1) + F_2 \sin(\theta + \phi_2) + F_3 \sin(\theta + \phi_3) \quad (\text{A-1})$$

Noting that

$$\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

Then

$$\begin{aligned} w &= F_1 \sin \theta \cos \phi_1 + F_1 \cos \theta \sin \phi_1 \\ &\quad + F_2 \sin \theta \cos \phi_2 + F_2 \cos \theta \sin \phi_2 \\ &\quad + F_3 \sin \theta \cos \phi_3 + F_3 \cos \theta \sin \phi_3 \end{aligned} \quad (\text{A-2})$$

or

$$\begin{aligned} w &= [F_1 \cos \phi_1 + F_2 \cos \phi_2 + F_3 \cos \phi_3] \sin \theta \\ &\quad + [F_1 \sin \phi_1 + F_2 \sin \phi_2 + F_3 \sin \phi_3] \cos \theta \end{aligned} \quad (\text{A-3})$$

Let

$$\begin{aligned} H_s &= F_1 \cos \phi_1 + F_2 \cos \phi_2 + F_3 \cos \phi_3 \\ H_c &= F_1 \sin \phi_1 + F_2 \sin \phi_2 + F_3 \sin \phi_3 \end{aligned} \quad (\text{A-4})$$

Then

$$w = H_s \sin \theta + H_c \cos \theta \quad (\text{A-5})$$

or

$$w = \sqrt{H_s^2 + H_c^2} \left\{ \left( \frac{H_s}{\sqrt{H_s^2 + H_c^2}} \right) \sin \theta + \left( \frac{H_c}{\sqrt{H_s^2 + H_c^2}} \right) \cos \theta \right\} \quad (\text{A-6})$$

or still

$$w = \sqrt{H_s^2 + H_c^2} \left\{ \cos \beta \sin \theta + \sin \beta \cos \theta \right\} \quad (\text{A-7})$$

where  $\beta$  is the phase angle defined by:

$$\beta = \arcsin \left\{ \frac{H_c}{\sqrt{H_s^2 + H_c^2}} \right\} \quad \text{or} \quad \beta = \tan^{-1} \left( \frac{H_c}{H_s} \right) \quad (\text{A-8})$$

and

$$w = \sqrt{H_s^2 + H_c^2} \sin(\theta + \beta) \quad (\text{A-9})$$

Now consider the general case where:

$$\begin{aligned} w &= F_1 \sin(\theta + \phi_{s1}) + F_2 \sin(\theta + \phi_{s2}) + \dots + F_L \sin(\theta + \phi_{sL}) \\ &\quad + G_1 \cos(\theta + \phi_{c1}) + G_2 \cos(\theta + \phi_{c2}) + \dots + G_M \cos(\theta + \phi_{cM}) \end{aligned} \quad (\text{A-10})$$

Noting that

$$\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

and

$$\cos(\theta + \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$$

Then

$$\begin{aligned} w = & F_1 \cos \phi_{s1} \sin \theta + F_1 \sin \phi_{s1} \cos \theta + F_2 \cos \phi_{s2} \sin \theta + F_2 \sin \phi_{s2} \cos \theta \\ & + \dots + F_L \cos \phi_{sL} \sin \theta + F_L \sin \phi_{sL} \cos \theta \\ & + G_1 \cos \phi_{c1} \cos \theta - G_1 \sin \phi_{c1} \sin \theta + G_2 \cos \phi_{c2} \cos \theta - G_2 \sin \phi_{c2} \sin \theta \\ & + \dots + G_M \cos \phi_{cM} \cos \theta - G_M \sin \phi_{cM} \sin \theta \end{aligned} \quad (\text{A-11})$$

Equation (70) can be rewritten as

$$w = H_s \sin \theta + H_c \cos \theta \quad (\text{A-12})$$

where

$$\begin{aligned} H_s = & F_1 \cos \phi_{s1} + F_2 \cos \phi_{s2} + \dots + F_L \cos \phi_{sL} \\ & - G_1 \sin \phi_{c1} - G_2 \sin \phi_{c2} - \dots - G_M \sin \phi_{cM} \\ H_c = & F_1 \sin \phi_{s1} + F_2 \sin \phi_{s2} + \dots + F_L \sin \phi_{sL} \\ & + G_1 \cos \phi_{c1} + G_2 \cos \phi_{c2} + \dots + G_M \cos \phi_{cM} \end{aligned} \quad (\text{A-13})$$

Equation (71) can be rewritten as

$$w = \sqrt{H_s^2 + H_c^2} \left\{ \left( \frac{H_s}{\sqrt{H_s^2 + H_c^2}} \right) \sin \theta + \left( \frac{H_c}{\sqrt{H_s^2 + H_c^2}} \right) \cos \theta \right\} \quad (\text{A-14})$$

or still

$$w = \sqrt{H_s^2 + H_c^2} \left\{ \cos \beta \sin \theta + \sin \beta \cos \theta \right\} \quad (\text{A-15})$$

where  $\beta$  is the phase angle defined by:

$$\beta = \arcsin \left\{ \frac{H_c}{\sqrt{H_s^2 + H_c^2}} \right\} \quad \text{or} \quad \beta = \tan^{-1} \left( \frac{H_c}{H_s} \right) \quad (\text{A-16})$$

and

$$w = \sqrt{H_s^2 + H_c^2} \sin(\theta + \beta) \quad (\text{A-17})$$



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13. ABSTRACT (Maximum 200 words)  An important class of techniques to reduce helicopter vibration is based on using a Higher Harmonic controller to optimally define the Higher Harmonic blade pitch. These techniques typically require solution of a general optimization problem requiring the determination of a control vector which minimizes a performance index where functions of the control vector are subject to inequality constraints. Six possible constraint functions associated with swashplate blade pitch control were identified and defined. These functions constrain: 1) blade pitch Fourier Coefficients expressed in the Rotating System, 2) blade pitch Fourier Coefficients expressed in the Nonrotating System, 3) stroke of the individual actuators expressed in the Nonrotating System, 4) blade pitch expressed as a function of blade azimuth and actuator stroke, 5) time rate-of-change of the aforementioned parameters, and 6) required actuator power. The aforementioned constraints and the associated kinematics of swashplate blade pitch control by means of the strokes of the individual actuators are documented herein.				
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